ELASTICITY

1. **Rigid Body :**
A body whose size and shape cannot be changed, however large the applied force may be is called rigid body.
There is no perfectly rigid body in nature.

2. **Deformation force :**
The force which changes the size or shape or both of a body without moving it as a whole is called deformation force.

3. **Restoring force :**
The force which restores the size and shape of the body when deformation force is removed is called restoring force.
Magnitude of restoring force is equal to the deformation force. But they are in opposite direction. They do not form action, reaction pair. This force is responsible for the elastic nature of the
body.

4. **Elasticity :**
The property of a material by virtue of which it regains its original size and shape when deformation force is removed is called elasticity.
Ex : Steel, Rubber.
No body is perfectly elastic, but quartz is the nearest example.
Elasticity is molecular property of matter.

5. **Plasticity :**
The property of a material by virtue of which it does not regain the size and shape when the de formation force is removed is called Plasticity.
Ex : Putty dough, Chewing gum, Soldering lead
No body is perfectly plastic but putty is nearest example.

6. **Stress :**
The restoring force per unit area is called stress.

$$\text{Stress} = \frac{\text{restoring force}}{\text{area of cross section}} = \frac{F}{A}$$

Unit : N/m^2 or Pascal.

a. If the stress is normal to the surface, it is called normal stress. If the stress is tangential to the surface, it is called tangential stress.

$$\text{Normal stress} = \frac{\text{Normal restoring force}}{\text{Area of cross section}}$$

$$\text{Shearing stress} = \frac{\text{Tangential restoring force}}{\text{Area of cross section}}$$

b. When normal stress changes the volume of the body then it is called Volume Stress or Bulk Stress. It is denoted by 'P'.

c. Longitudinal stress is called tensile stress when there is an increase in length and compresive stress when there is at decrease in length.

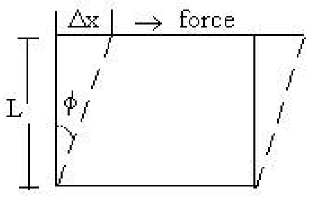
d. Longitudinal stress and bulk stress are normal stresses which produce change in size, shearing stress is a tangential stress which produce change in shape.

7. **Strain :**
The change produced per unit dimension is called strain.

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{e}{\ell}$$

Shearing strain:-



$$\text{Shearing strain} = \frac{\text{relative displacement between two layers}}{\text{distance between the layers}}$$

$$= \frac{\Delta x}{L} = \phi$$

This strain is due to the change in shape of the body.

Volumetric strain (or) Bulk strain

$$= \frac{\text{change in volume}}{\text{original volume}} = \frac{-\Delta V}{V}$$

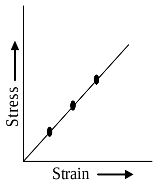
Negative sign indicates decrease in volume

8. **Elastic limit :**
The maximum value of the stress with in which the body regains its original size and shape is
called elastic limit.

9. **Hooke's law :**
With in the elastic limit, stress is directly proportional to strain.

$$\frac{\text{Stress}}{\text{Strain}} = E$$

With in the elastic limit, stress-strain graph is a straight line passing through the origin.



Slope of the graph is E.

10. **Young's modulus (Y) :**

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F\ell}{Ae}$$

If load attached to the wire is M, then F = Mg, and $A = \pi r^2$

$$Y = \frac{Mg\ell}{\pi r^2 e}$$

11. **Rigidity modulus :**
With in the elastic limit, the ratio of tangential stress to the shearing strain is called rigidity modulus.

$$\text{Rigidity modulus} = \frac{\text{tangential stress}}{\text{shearing strain}}$$

$$\Rightarrow \eta = \frac{F}{A\theta}$$

If η is small for a wire, it can be twisted easily.

12. **Bulk modulus (B)**
With in the elastic limit the ratio between volume stress and bulk strain is called bulk modulus.

$$\text{Bulk modulus} = \frac{\text{volume stress}}{\text{bulk strain}}$$

$$B = \frac{\frac{F}{A}}{\frac{-\Delta V}{V}} = -\frac{V}{\Delta V}\frac{F}{A} = \frac{-PV}{\Delta V}$$

(- sign shows decrease in volume)

13. The reciprocal of bulk modulus is called compressibility $K = \frac{1}{B}$

14. **Poisson's ratio (σ)**
The ratio lateral contraction strain to the longitudinal elongation strain is called Poisson'sratio.

$$\sigma = \frac{\text{lateral contraction strain}}{\text{longitudinal elongation strain}}$$

$$= \frac{\text{transverse strain}}{\text{longitudinal strain}} = \frac{\Delta r / r}{\Delta\ell / \ell} = \frac{\Delta r \ell}{\Delta\ell \times r}$$

i) As it is a ratio, it has no units and dimension.
ii) Theoretical limits of $\sigma = -1$ to 0.5
Practical limits of $\sigma = 0$ to 0.5
iii) For an incompressible substance $\sigma = 0.5$

15. **Thermal force :**
When a metal bar is fixed between two supports and heated, it tries to expand and exerts force on the walls. This is called thermal force.

$$F = AY\alpha\theta$$

Thermal force is independent of length of the bar.
Thermal stress (linear compressive stress)

$$\frac{\text{force}}{\text{area}} = \frac{AY\alpha\theta}{A} = Y\alpha\theta.$$

16. The work done in stretching a wire / elastic potential energy in a stretched wire

$$W = \frac{1}{2}\text{Stress x Strain x Volume}$$

Potential

$$W = \frac{1}{2}Fe = \frac{1}{2}\frac{e^2 AY}{\ell}$$

17. Strain energy per unit volume (energy density)

$$E = \frac{1}{2}\text{ x stress x strain } = \frac{1}{2}Y(Strain)^2$$

Also, $E = \frac{(Stress)^2}{2Y}\left[\therefore Y = \frac{Stress}{Strain}\right]$